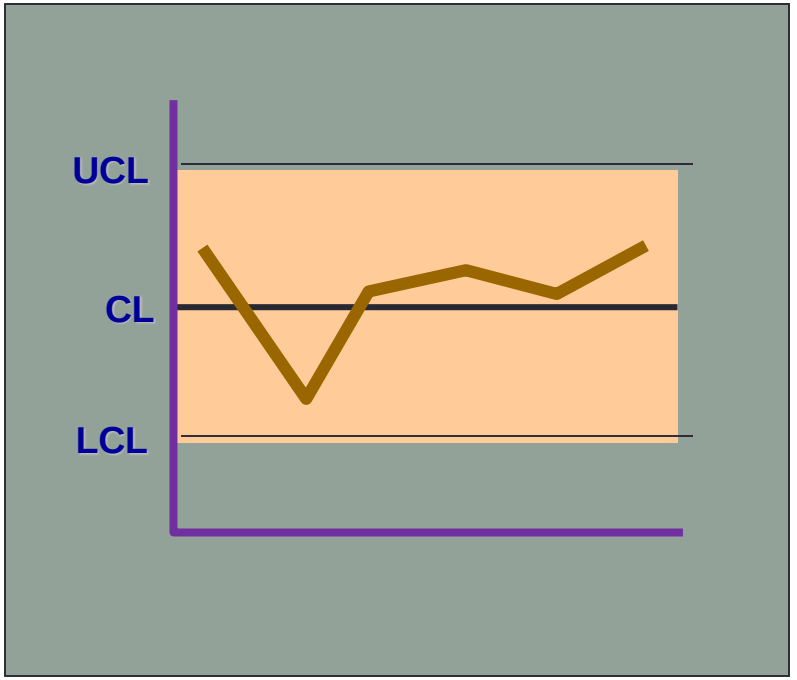
VARIABLE CONTROL CHART

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Professor
Department of Industrial and Management Engineering

All figures are taken from(unless otherwise mentioned)
Introduction to Statistical process Control
Douglas. C Montgomery
6th Edition

Basics of Statistical Process Control

- Statistical Process Control (SPC)
  - monitoring production process to **detect** and **prevent** poor quality
- Sample
  - **subset of items** produced to use for inspection
- Control Charts
  - process is **within statistical control** limits



Variability

- Variability is inherent in every process
  - Natural or common causes
  - Special or assignable causes
- Provides a statistical signal when assignable causes are present
- Detect and eliminate assignable causes of variation

Variability

- **Random**
  - common causes
  - inherent in a process
  - can be eliminated only through improvements in the system
- **Non-Random**
  - special causes
  - due to identifiable factors
  - can be modified through operator or management action

Random or Natural Variations

- Natural variations in the production process
- These are to be expected
- Output measures follow a probability distribution
- For any distribution there is a measure of central tendency and dispersion

Non-Random or Assignable Variations

- Variations that can be traced to a specific reason (machine wear, misadjusted equipment, fatigued or untrained workers)
- The objective is to discover when assignable causes are present and eliminate them

Quality Measures

Attribute

- a product characteristic that can be evaluated with a discrete response
- good – bad; yes – no

Variable

- a product characteristic that is continuous and can be measured
- weight, length, …..

Types of Data

Variables

- Characteristics that can take any real value
- May be in whole or in fractional numbers
- Continuous random variables

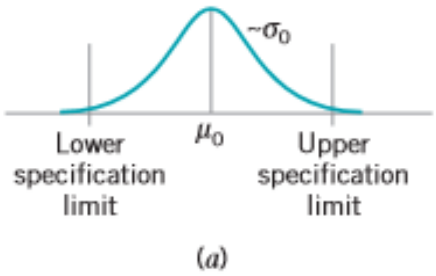
Attributes

- Defect-related characteristics
- Classify products as either good or bad or count defects
- Categorical or discrete random variables

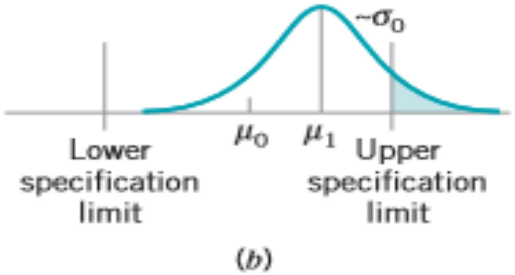
Control Charts for Variables

- For variables that have continuous dimensions
  - weight, speed, length, strength, etc.
- $\bar{x}$-charts are to control the central tendency of the process
- R-charts are to control the dispersion of the process

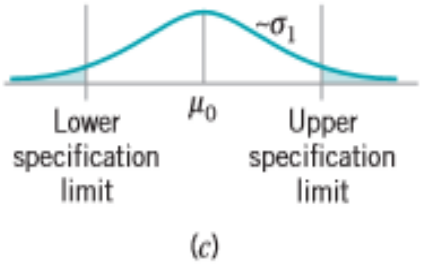
Need for controlling mean and variability



Mean and Standard deviation at nominal level



Change in process mean



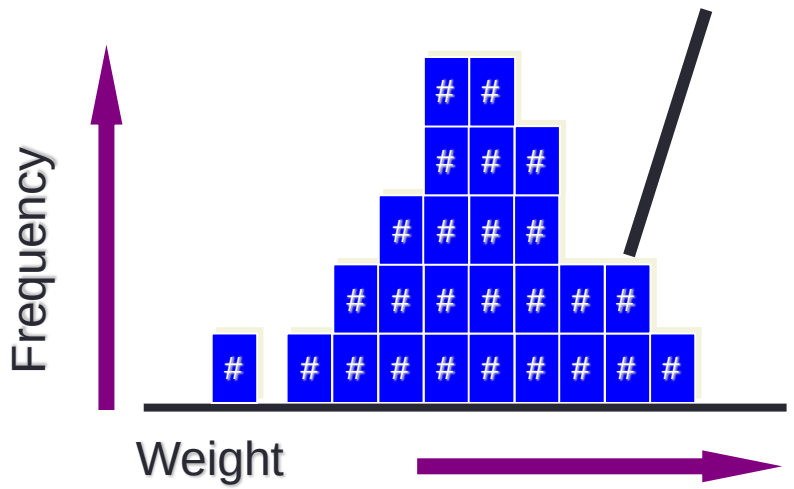
Change in process SD

Samples

To measure the process, we take samples and analyze the sample statistics following these steps

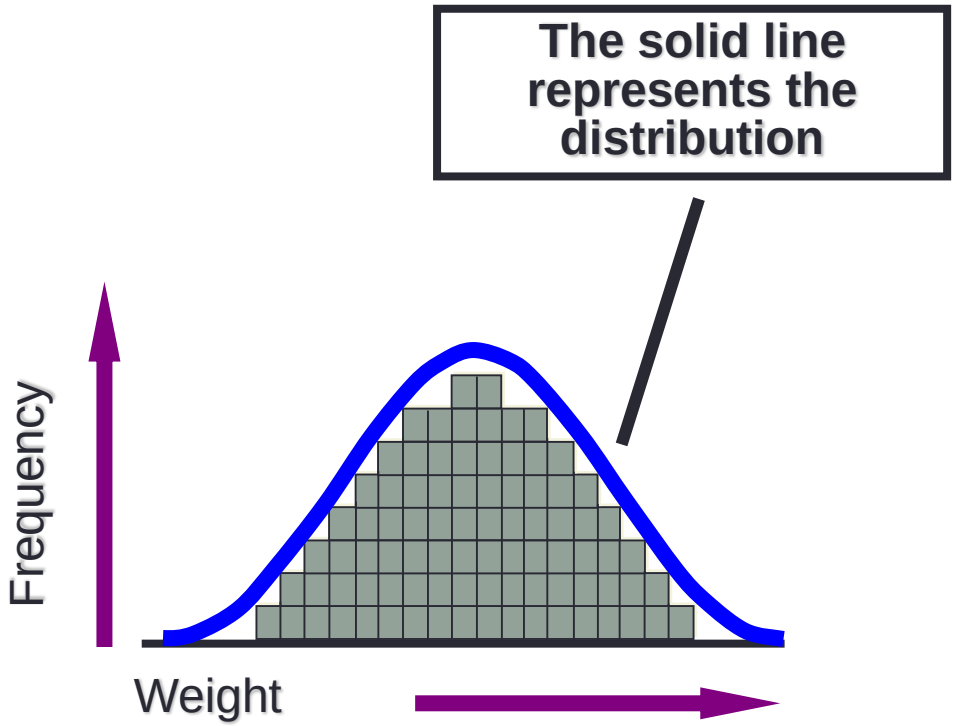
(a) Samples of the product, say five boxes taken off the machine line, vary from each other in weight

Each of these represents one sample of five boxes



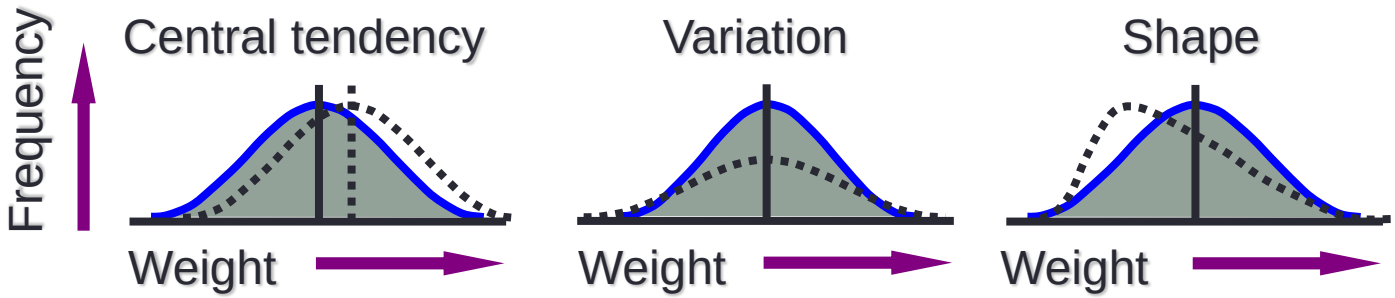
Samples

(b) After enough samples are taken from a stable process, they form a pattern called a **distribution**



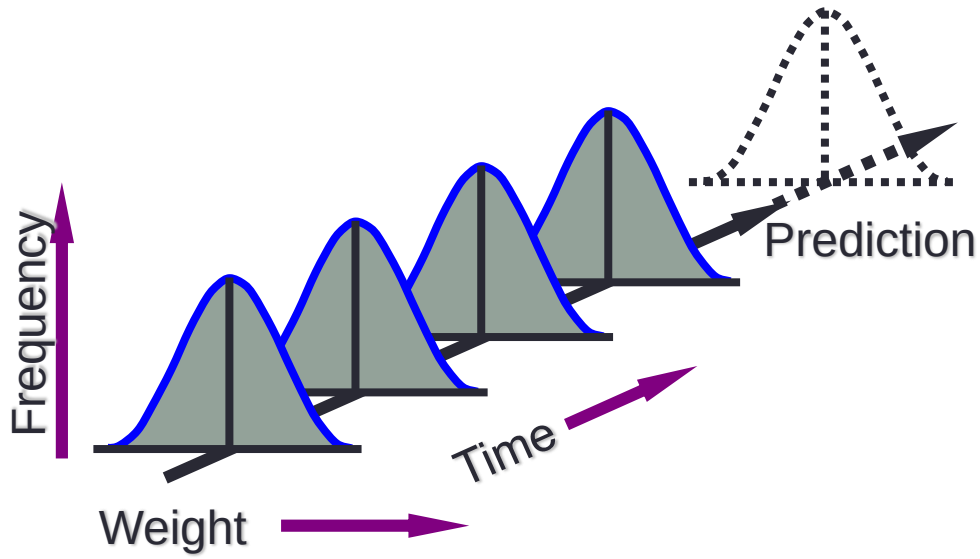
Samples

(c)There are many types of distributions, including the normal (bell-shaped) distribution, but distributions do differ in terms of central tendency (mean), standard deviation or variance, and shape

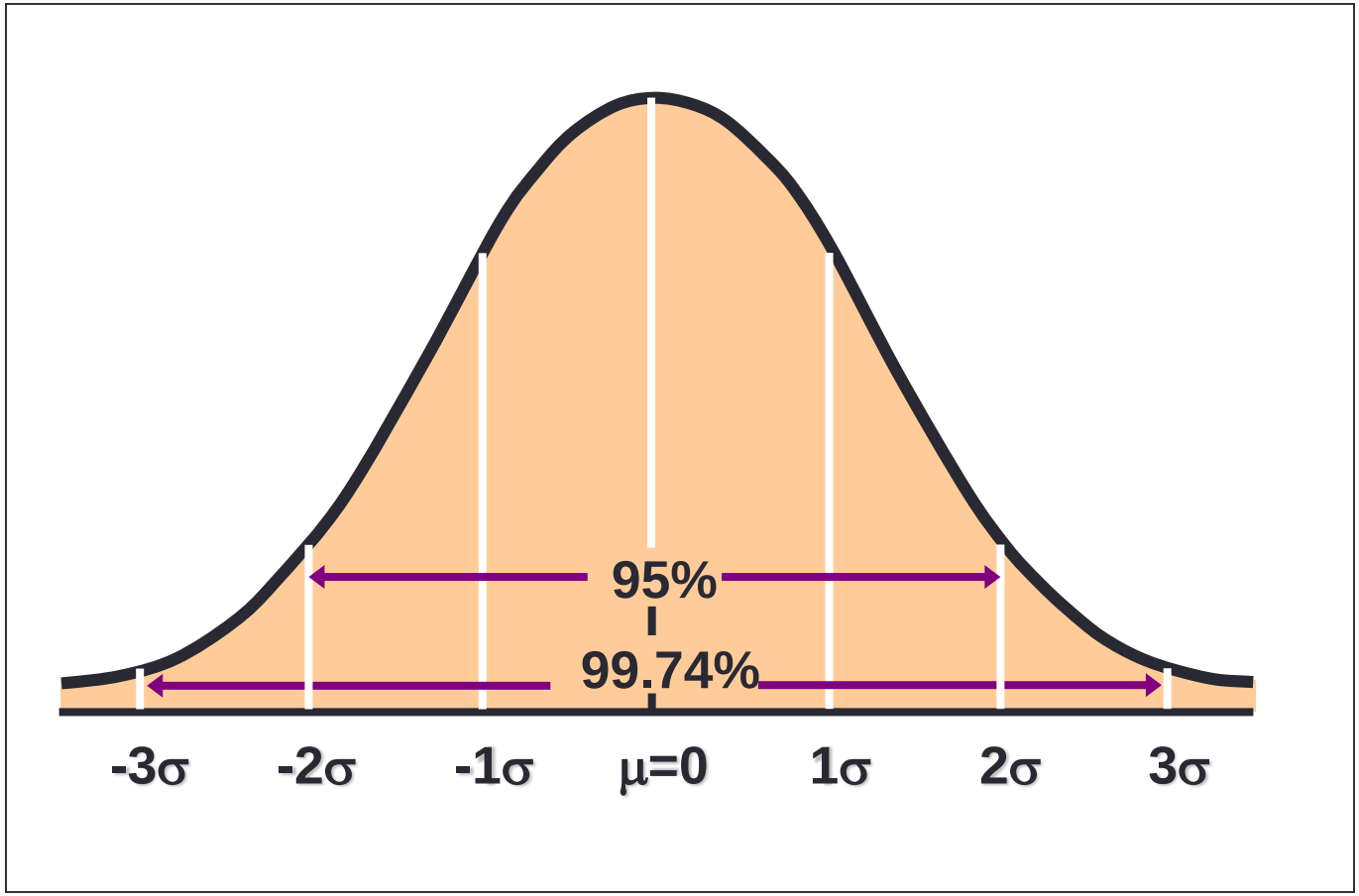


Samples

(d)If only natural causes of variation are present, the output of a process forms a distribution that is stable over time and is predictable



Normal Distribution



Basis of $\bar{x}$ and R chart

- Let a quality characteristic is normally distributed with mean μ and standard deviation σ
  - If $x_1, x_2 \ldots\ldots x_n$ is a sample of size n, then sample mean is given by

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

  - With a probability of 1-α, any sample mean will lie between

$$\mu + Z_{\alpha/2}\sigma_{\bar{x}} = \mu + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \quad \text{and} \quad \mu - Z_{\alpha/2}\sigma_{\bar{x}} = \mu - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

  - Using the above two equations we can find the central and control lines
- However in real life we will not know μ or σ

Estimating μ and σ

- Suppose we take m samples each of size n
  - The best estimate of process average is given as

$$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \cdots + \bar{x}_m}{m}$$

  - Here $\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_m$ represents the mean of each sample
  - To estimate the σ, we measure the range for the m samples
  - Range of each sample :
  - Average range for all samples:

$$R = x_{\max} - x_{\min}$$

$$\bar{R} = \frac{R_1 + R_2 + \cdots + R_m}{m}$$

Control Limits for the charts

Control Limits for the $\bar{x}$ Chart

$$\text{UCL} = \bar{\bar{x}} + A_2\bar{R}$$

$$\text{Center line} = \bar{\bar{x}}$$

$$\text{LCL} = \bar{\bar{x}} - A_2\bar{R}$$

Control Limits for the R Chart

$$\text{UCL} = D_4\bar{R}$$

$$\text{Center line} = \bar{R}$$

$$\text{LCL} = D_3\bar{R}$$

Reference Charts

Observations in Sample, n	Chart for Averages: Factors for Control Limits A	A_2	A_3	Chart for Averages: Factors for Center Line c_4	$1/c_4$	Chart for Standard Deviations: Factors for Control Limits B_3	B_4	B_5	B_6
2	2.121	1.880	2.659	0.7979	1.2533	0	3.267	0	2.606
3	1.732	1.023	1.954	0.8862	1.1284	0	2.568	0	2.276
4	1.500	0.729	1.628	0.9213	1.0854	0	2.266	0	2.088
5	1.342	0.577	1.427	0.9400	1.0638	0	2.089	0	1.964
6	1.225	0.483	1.287	0.9515	1.0510	0.030	1.970	0.029	1.874
7	1.134	0.419	1.182	0.9594	1.0423	0.118	1.882	0.113	1.806
8	1.061	0.373	1.099	0.9650	1.0363	0.185	1.815	0.179	1.751
9	1.000	0.337	1.032	0.9693	1.0317	0.239	1.761	0.232	1.707
10	0.949	0.308	0.975	0.9727	1.0281	0.284	1.716	0.276	1.669
11	0.905	0.285	0.927	0.9754	1.0252	0.321	1.679	0.313	1.637
12	0.866	0.266	0.886	0.9776	1.0229	0.354	1.646	0.346	1.610
13	0.832	0.249	0.850	0.9794	1.0210	0.382	1.618	0.374	1.585
14	0.802	0.235	0.817	0.9810	1.0194	0.406	1.594	0.399	1.563
15	0.775	0.223	0.789	0.9823	1.0180	0.428	1.572	0.421	1.544
16	0.750	0.212	0.763	0.9835	1.0168	0.448	1.552	0.440	1.526
17	0.728	0.203	0.739	0.9845	1.0157	0.466	1.534	0.458	1.511
18	0.707	0.194	0.718	0.9854	1.0148	0.482	1.518	0.475	1.496
19	0.688	0.187	0.698	0.9862	1.0140	0.497	1.503	0.490	1.483
20	0.671	0.180	0.680	0.9869	1.0133	0.510	1.490	0.504	1.470
21	0.655	0.173	0.663	0.9876	1.0126	0.523	1.477	0.516	1.459
22	0.640	0.167	0.647	0.9882	1.0119	0.534	1.466	0.528	1.448
23	0.626	0.162	0.633	0.9887	1.0114	0.545	1.455	0.539	1.438
24	0.612	0.157	0.619	0.9892	1.0109	0.555	1.445	0.549	1.429
25	0.600	0.153	0.606	0.9896	1.0105	0.565	1.435	0.559	1.420

Reference Charts

Observations in Sample, n	Factors for Center Line		Chart for Ranges: Factors for Control Limits				
	d_2	$1/d_2$	d_3	D_1	D_2	D_3	D_4
2	1.128	0.8865	0.853	0	3.686	0	3.267
3	1.693	0.5907	0.888	0	4.358	0	2.574
4	2.059	0.4857	0.880	0	4.698	0	2.282
5	2.326	0.4299	0.864	0	4.918	0	2.114
6	2.534	0.3946	0.848	0	5.078	0	2.004
7	2.704	0.3698	0.833	0.204	5.204	0.076	1.924
8	2.847	0.3512	0.820	0.388	5.306	0.136	1.864
9	2.970	0.3367	0.808	0.547	5.393	0.184	1.816
10	3.078	0.3249	0.797	0.687	5.469	0.223	1.777
11	3.173	0.3152	0.787	0.811	5.535	0.256	1.744
12	3.258	0.3069	0.778	0.922	5.594	0.283	1.717
13	3.336	0.2998	0.770	1.025	5.647	0.307	1.693
14	3.407	0.2935	0.763	1.118	5.696	0.328	1.672
15	3.472	0.2880	0.756	1.203	5.741	0.347	1.653
16	3.532	0.2831	0.750	1.282	5.782	0.363	1.637
17	3.588	0.2787	0.744	1.356	5.820	0.378	1.622
18	3.640	0.2747	0.739	1.424	5.856	0.391	1.608
19	3.689	0.2711	0.734	1.487	5.891	0.403	1.597
20	3.735	0.2677	0.729	1.549	5.921	0.415	1.585
21	3.778	0.2647	0.724	1.605	5.951	0.425	1.575
22	3.819	0.2618	0.720	1.659	5.979	0.434	1.566
23	3.858	0.2592	0.716	1.710	6.006	0.443	1.557
24	3.895	0.2567	0.712	1.759	6.031	0.451	1.548
25	3.931	0.2544	0.708	1.806	6.056	0.459	1.541

Unbiased estimator of σ

- If $\overline{R}$ is the average range of m samples, then to estimate sigma, we use

- This is an unbiased estimator of σ

$$\hat{\sigma} = \frac{\overline{R}}{d_2}$$

Phase 1 of Control Chart Usage

- Since at the beginning, the chart uses preliminary samples, the control limits are treated as trial control limits
- To check if process was in control during the initial m samples plot he charts and look for any pattern or any point outside the control limit
- If there are points outside control limits
  - Revise the control limits
- Check assignable cause for each of the points outside control limits
- If there exists an assignable cause, discard the point and recalculate control limits with the remaining points

Phase 1 of Control Chart Usage

- If no assignable cause if found
  - Discard the point without much justification
  - Retain the point and see in future if process remains in control
- If initially a lot of points remain out of control
  - Dropping too many reduce available data points
  - Inspecting causes for all points may not be possible/worthwhile
- In such cases look for patterns in control chart
- Identification and removal of process problem causes major process improvement

An example

A hard-bake process (see Section 5.3.1) is used in conjunction with photolithography in semiconductor manufacturing. We wish to establish statistical control of the flow width of the resist in this process using $\bar{x}$ and R charts. Twenty-five samples, each of size five wafers, have been taken when we think the process is in control. The interval of time between samples or subgroups is one hour. The flow width measurement data (in microns) from these samples are shown in Table 6.1.

Flow Width Measurements (microns) for the Hard-Bake Process

Sample Number	Wafers						
	1	2	3	4	5	$\bar{x}_i$	R_i
1	1.3235	1.4128	1.6744	1.4573	1.6914	1.5119	0.3679
2	1.4314	1.3592	1.6075	1.4666	1.6109	1.4951	0.2517
3	1.4284	1.4871	1.4932	1.4324	1.5674	1.4817	0.1390
4	1.5028	1.6352	1.3841	1.2831	1.5507	1.4712	0.3521
5	1.5604	1.2735	1.5265	1.4363	1.6441	1.4882	0.3706
6	1.5955	1.5451	1.3574	1.3281	1.4198	1.4492	0.2674
7	1.6274	1.5064	1.8366	1.4177	1.5144	1.5805	0.4189
8	1.4190	1.4303	1.6637	1.6067	1.5519	1.5343	0.2447
9	1.3884	1.7277	1.5355	1.5176	1.3688	1.5076	0.3589
10	1.4039	1.6697	1.5089	1.4627	1.5220	1.5134	0.2658
11	1.4158	1.7667	1.4278	1.5928	1.4181	1.5242	0.3509
12	1.5821	1.3355	1.5777	1.3908	1.7559	1.5284	0.4204
13	1.2856	1.4106	1.4447	1.6398	1.1928	1.3947	0.4470
14	1.4951	1.4036	1.5893	1.6458	1.4969	1.5261	0.2422
15	1.3589	1.2863	1.5996	1.2497	1.5471	1.4083	0.3499
16	1.5747	1.5301	1.5171	1.1839	1.8662	1.5344	0.6823
17	1.3680	1.7269	1.3957	1.5014	1.4449	1.4874	0.3589
18	1.4163	1.3864	1.3057	1.6210	1.5573	1.4573	0.3153
19	1.5796	1.4185	1.6541	1.5116	1.7247	1.5777	0.3062
20	1.7106	1.4412	1.2361	1.3820	1.7601	1.5060	0.5240
21	1.4371	1.5051	1.3485	1.5670	1.4880	1.4691	0.2185
22	1.4738	1.5936	1.6583	1.4973	1.4720	1.5390	0.1863
23	1.5917	1.4333	1.5551	1.5295	1.6866	1.5592	0.2533
24	1.6399	1.5243	1.5705	1.5563	1.5530	1.5688	0.1156
25	1.5797	1.3663	1.6240	1.3732	1.6887	1.5264	0.3224
						$\Sigma\bar{x}_i = 37.6400$	$\Sigma R_i = 8.1302$
						$\bar{\bar{x}} = 1.5056$	$\bar{R} = 0.32521$

The R chart

- Central Line for R chart

$$\bar{R} = \frac{\sum_{i=1}^{25} R_i}{25} = \frac{8.1302}{25} = 0.32521$$

- Control Limits for the chart

$$\text{LCL} = \bar{R}D_3 = 0.32521(0) = 0$$
$$\text{UCL} = \bar{R}D_4 = 0.32521(2.114) = 0.68749$$

- R chart



- Process variability is in control

The $\bar{x}$ chart

- The central line

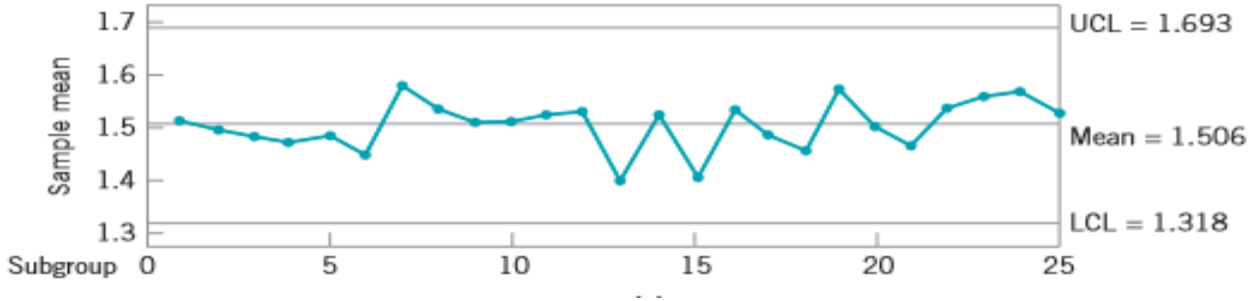
$$\bar{\bar{x}} = \frac{\sum_{i=1}^{25} \bar{x}_i}{25} = \frac{37.6400}{25} = 1.5056$$

- The control limits

$$UCL = \bar{\bar{x}} + A_2\bar{R} = 1.5056 + (0.577)(0.32521) = 1.69325$$

$$LCL = \bar{\bar{x}} - A_2\bar{R} = 1.5056 - (0.577)(0.32521) = 1.31795$$

- The $\bar{x}$ chart



- No point is out of control

Estimating Process Capability

- Estimating process standard deviation

$$\hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{0.32521}{2.326} = 0.1398 \text{ microns}$$

- Estimate of the fraction of nonconforming wafers produced
  - Assumption: Flow width is a normally distributed random variable with Mean=1.5056 and Std. Dev.=.1398

$$\begin{aligned} p &= P\{x < 1.00\} + P\{x > 2.00\} \\ &= \Phi\left(\frac{1.00 - 1.5056}{0.1398}\right) + 1 - \Phi\left(\frac{2.00 - 1.5056}{0.1398}\right) \\ &= \Phi(-3.61660) + 1 - \Phi(3.53648) \\ &\simeq 0.00015 + 1 - 0.99980 \\ &\simeq 0.00035 \end{aligned}$$

- About 0.035 percent [350 parts per million (ppm)] of the wafers produced will be outside of the specifications.

Estimating Process Capability Ratio

- Process Capability Ratio is expressed as:

$$C_p = \frac{\text{USL} - \text{LSL}}{6\sigma}$$

- For the above example

$$\hat{C}_p = \frac{2.00 - 1.00}{6(0.1398)} = \frac{1.00}{0.8388} = 1.192$$

- Another way to interpret the Process Capability Ratio

$$P = \left(\frac{1}{C_p}\right)100\%$$

  - the percentage of the specification band that the process uses up
  - For above example

$$\hat{P} = \left(\frac{1}{\hat{C}_p}\right)100\% = \left(\frac{1}{1.192}\right)100\% = 83.89$$

Revision of control limits and central lines

- Practitioners establish regular periods for review and revision of control chart limits, such as every week, every month, or every 25, 50, or 100 samples
  - highly desirable to use at least 25 samples
- Sometimes the user will replace the center line of the x chart with a target value
  - Helpful in shifting the process average to the desired value
  - Processes where the mean may be changed by a fairly simple adjustment of a variable that can be manipulated in the process
- Once a set of reliable control limits is established, we use the control chart for monitoring future production (Phase 2 operations of the charts)

Some Guidelines for Creating the Charts

- Rational Subgroup
  - The $\bar{x}$ chart monitors between-sample variability (variability in the process over time)
  - The R chart measures within-sample variability (the instantaneous process variability at a given time).
  - Rational subgroup concept means that subgroups or samples should be selected so that if assignable causes are present, the chance for differences between subgroups will be maximized, while the chance for differences due to these assignable causes within a subgroup will be minimized

Some guidelines …. continued

- To design the $\bar{x}$ and R charts, we must specify the sample size, control limit width, and frequency of sampling to be used

- If the $\bar{x}$ chart is used to detect moderate to large process shifts(2 σ or more) than small sample size is effective
  - For detecting small shifts, sample size of n=15 to 25 may be required

Some guidelines….continued

- The R chart is relatively insensitive to shifts in the process standard deviation for small samples.
- samples of size n = 5 have only about a 40% chance of detecting on the first sample a shift in the process standard deviation from s to 2s
- Consequently, for large n—say, n>10 or 12—it is probably best to use a control chart for s or s2 instead of the R chart
- The problem of choosing the sample size and the frequency of sampling is one of allocating sampling effort

R example

6.26. A TiW layer is deposited on a substrate using a sputtering tool. Table 6E.13 contains layer thickness measurements (in Angstroms) on 20 subgroups of four substrates.

(a) Set up $\bar{x}$ and R control charts on this process. Is the process in control? Revise the control limits as necessary.

(b) Estimate the mean and standard deviation of the process.

(c) Is the layer thickness normally distributed?

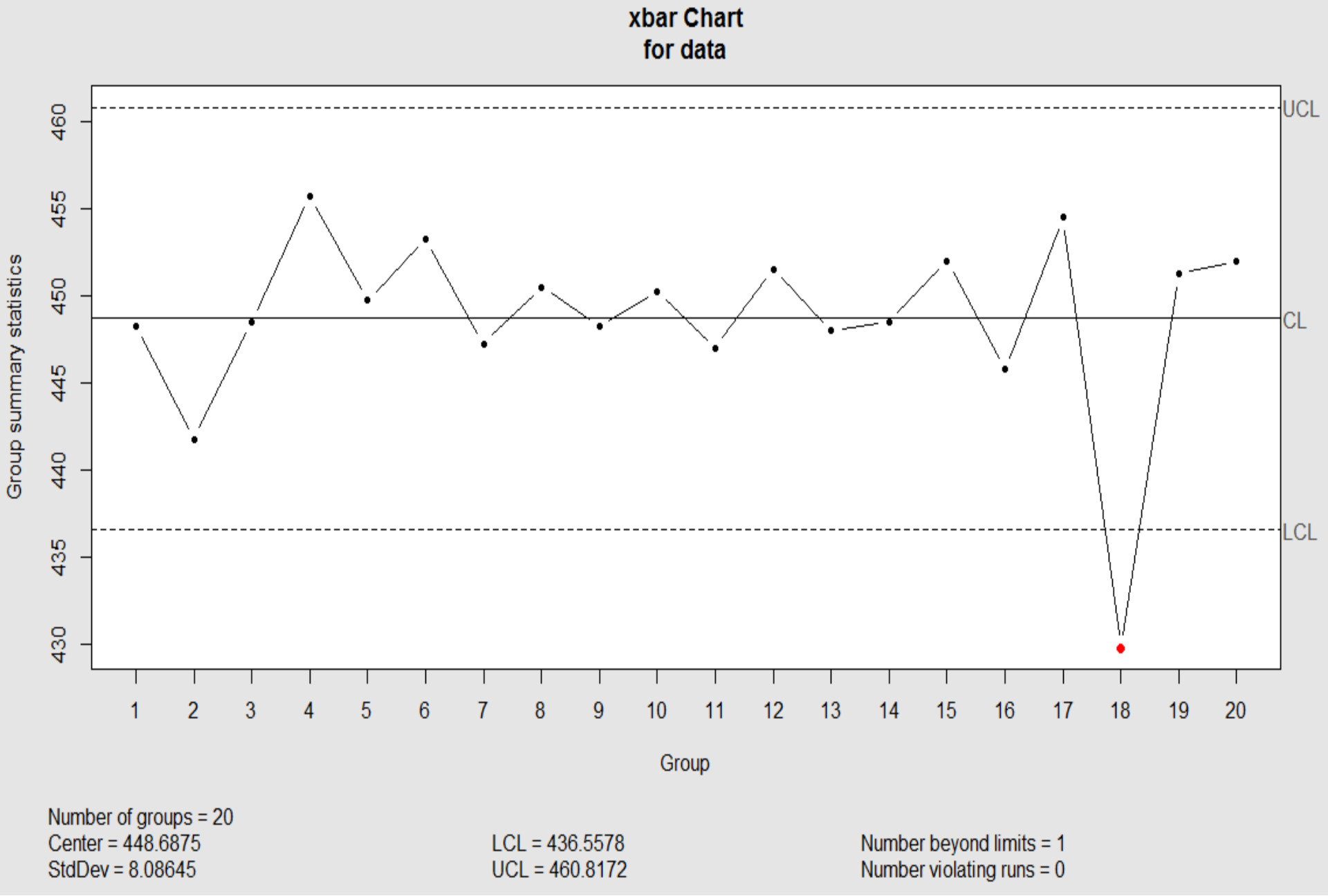
(d) If the specifications are at 450 ± 30, estimate the process capability.

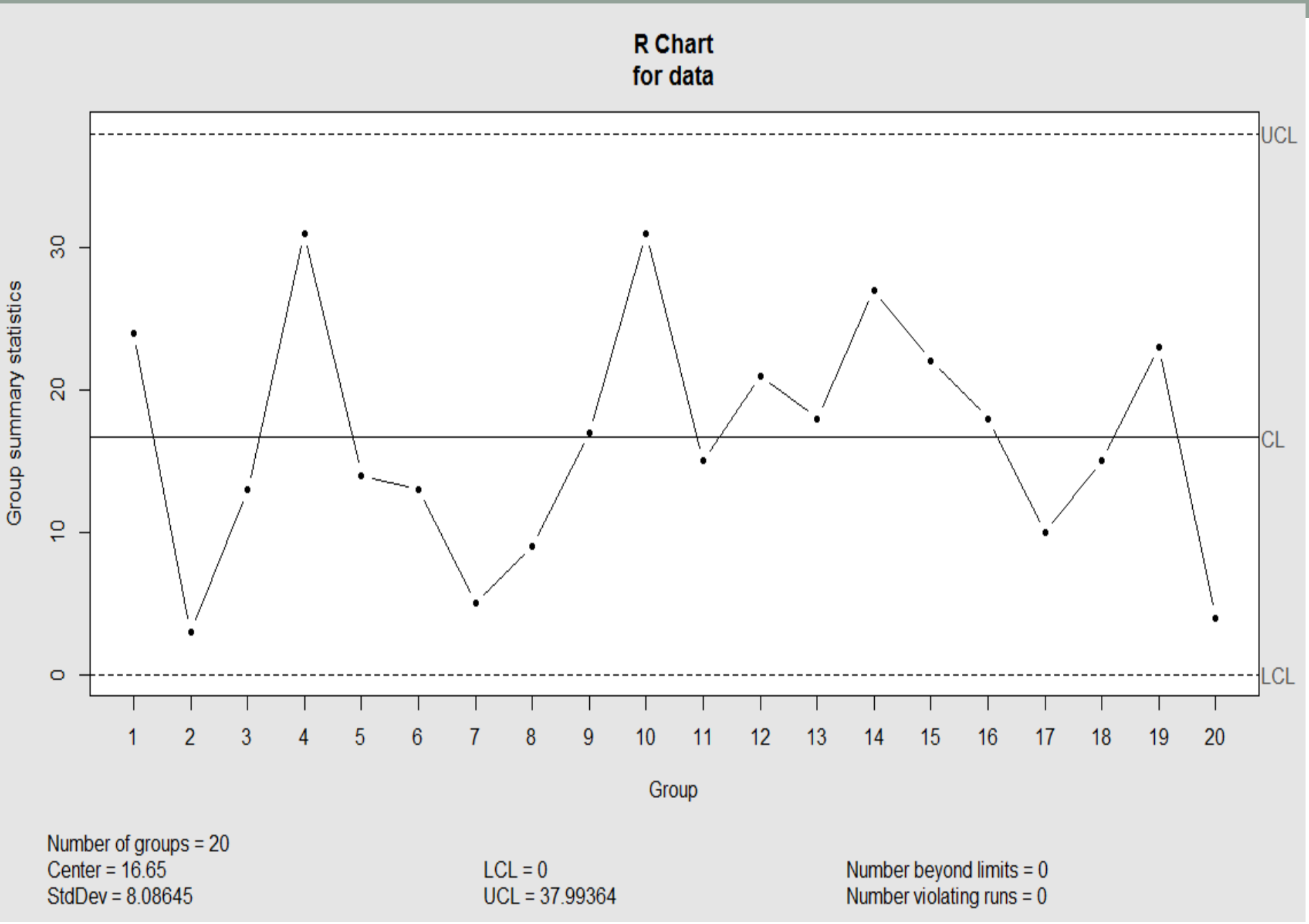
TABLE 6E.13
Layer Thickness Data for Exercise 6.26

Subgroup	x_1	x_2	x_3	x_4
1	459	449	435	450
2	443	440	442	442
3	457	444	449	444
4	469	463	453	438
5	443	457	445	454
6	444	456	456	457
7	445	449	450	445
8	446	455	449	452
9	444	452	457	440
10	432	463	463	443
11	445	452	453	438
12	456	457	436	457
13	459	445	441	447
14	441	465	438	450
15	460	453	457	438
16	453	444	451	435
17	451	460	450	457
18	422	431	437	429
19	444	446	448	467
20	450	450	454	454

R code

- data<-matrix(nrow=20,ncol=6)
- data[,1]=c(459,443,457,469,443,444,445,446,444,432,445,456,459,441,460,453,451,422,444,450)
- data[,2]=c(449,440,444,463,457,456,449,455,452,463,452,457,445,465,453,444,460,431,446,450)
- data[,3]=c(435,442,449,453,445,456,450,449,457,463,453,436,441,438,457,451,450,437,448,454)
- data[,4]=c(450,442,444,438,454,457,445,452,440,443,438,457,447,450,438,435,457,429,467,454)
- library(qcc)
  - # Here we use the package qcc
  - # The package has to be installed first before using
- i<-qcc(data,type='xbar')
- j<-qcc(data,type="R")


xbar Chart
for data
Group summary statistics
Group
UCL
CL
LCL
460
455
450
445
440
435
430
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
Number of groups = 20
Center = 448.6875
StdDev = 8.08645
LCL = 436.5578
UCL = 460.8172
Number beyond limits = 1
Number violating runs = 0

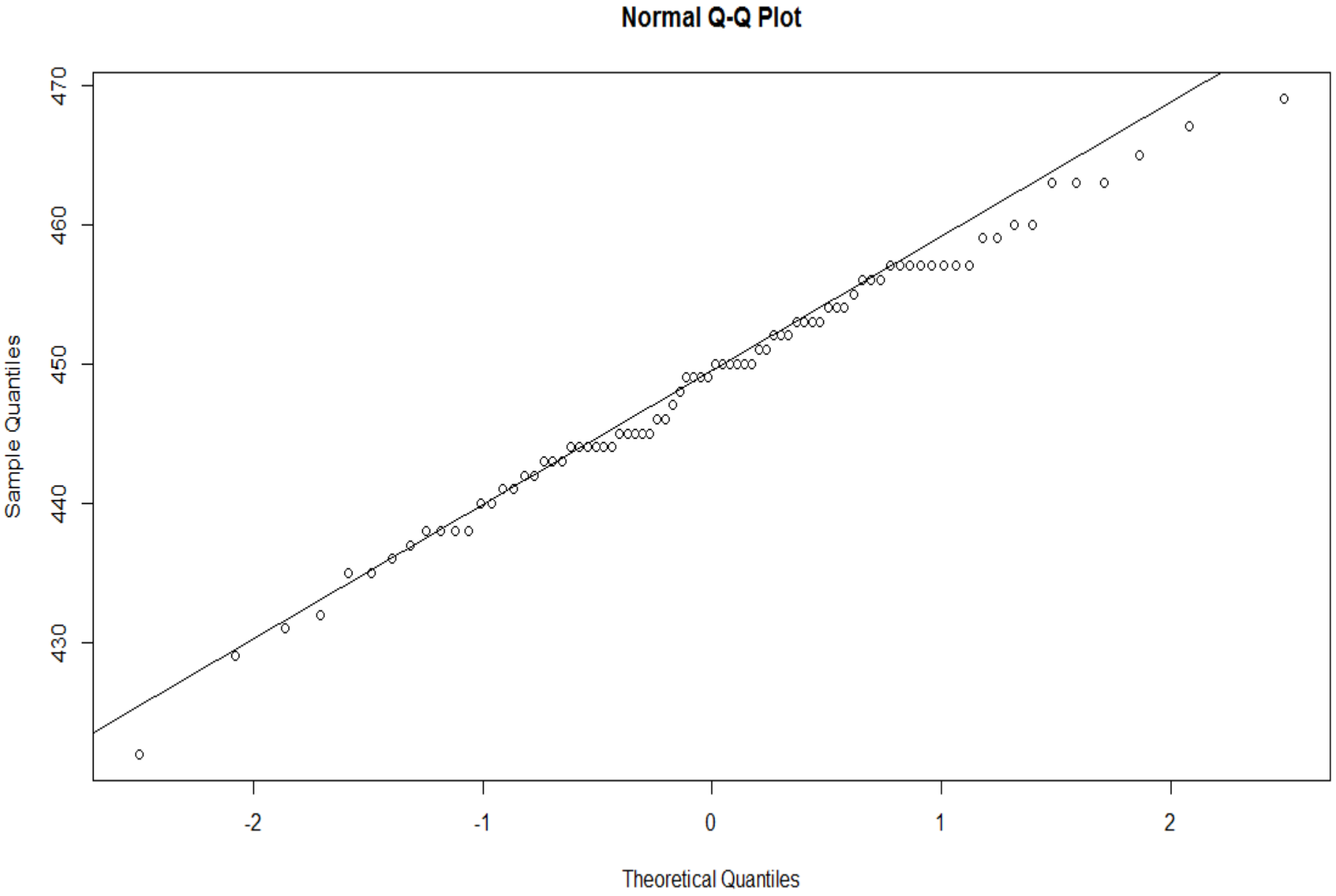

R Chart
for data
Group summary statistics
0
10
20
30
UCL
CL
LCL
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
Group
Number of groups = 20
Center = 16.65
StdDev = 8.08645
LCL = 0
UCL = 37.99364
Number beyond limits = 0
Number violating runs = 0

Example - Continued

- As can be seen from x bar chart, the third last point is out of control
- We remove to point to recalculate the control limits of both the charts
  - In real life we do this only after finding some assignable cause
- i<-qcc(data[-18,],type="xbar")
- j<-qcc(data[-18,],type="R")
- # if you type i$ in Rstudio, it will give you the available parameters like std.dev.,Center etc.
- Mean can be seen fro the charts
  - We will estimate the standard deviation after 2 slides

Example continued

- To check normality, we draw the q-q plot or the quantile-quantile plot
- A Q-Q plot is a scatterplot created by plotting two sets of quantiles against one another. If both sets of quantiles came from the same distribution, we should see the points forming a line that’s roughly straight.
- In checking normality, one quantile is taken from normal to see if the other data set is also normal.
- qqnorm(z)
- qqline(z)


Normal Q-Q Plot
Sample Quantiles
Theoretical Quantiles
430
440
450
460
470
-2
-1
0
1
2

Example continued

- Checking process capability:
  - First we estimate the standard deviation

$$\hat{\sigma} = \frac{\bar{R}}{d_2}$$

  - Here estimated σ=16.65/2.059 (check from table for n=4)
  - So estimated σ=8.08645
    - For these calculations we use the values that includes the 18th point
  - Now process capability

$$C_p = \frac{\text{USL} - \text{LSL}}{6\sigma}$$

  - $C_{p=}$ 60/(6*8.08)=1.2376

Variable Sample Size

- When a permanent change is required for the sample size, the limits are to be recalculated
- Let

$\overline{R}_{old}$ = average range for the old sample size
$\overline{R}_{new}$ = average range for the new sample size
n_{old} = old sample size
n_{new} = new sample size
$d_2(\text{old})$ = factor d_2 for the old sample size
$d_2(\text{new})$ = factor d_2 for the new sample size

New control limits

- For $\bar{x}$ chart

$$\text{UCL} = \bar{\bar{x}} + A_2\left[\frac{d_2(\text{new})}{d_2(\text{old})}\right]\bar{R}_{\text{old}}$$

$$\text{LCL} = \bar{\bar{x}} - A_2\left[\frac{d_2(\text{new})}{d_2(\text{old})}\right]\bar{R}_{\text{old}}$$

  - The central line $\bar{\bar{x}}$ is unchanged
- For R chart

$$\text{UCL} = D_4\left[\frac{d_2(\text{new})}{d_2(\text{old})}\right]\bar{R}_{\text{old}}$$

$$\text{CL} = \bar{R}_{\text{new}} = \left[\frac{d_2(\text{new})}{d_2(\text{old})}\right]\bar{R}_{\text{old}}$$

$$\text{LCL} = \max\left\{0, D_3\left[\frac{d_2(\text{new})}{d_2(\text{old})}\right]\bar{R}_{\text{old}}\right\}$$

Example

To illustrate the above procedure, consider the $\bar{x}$ and R charts developed for the hard-bake process in Example 6.1. These charts were based on a sample size of five wafers. Suppose that since the process exhibits good control, the process engineering personnel want to reduce the sample size to three wafers. Set up the new control charts.

$$n_{\text{old}} = 5 \qquad \bar{R}_{\text{old}} = 0.32521 \qquad d_2(\text{old}) = 2.326 \qquad d_2(\text{new}) = 1.693$$

For $\bar{x}$ chart the calculations are

$$\text{UCL} = \bar{\bar{x}} + A_2\left[\frac{d_2(\text{new})}{d_2(\text{old})}\right]\bar{R}_{\text{old}}$$

$$= 1.5056 + (1.023)\left[\frac{1.693}{2.326}\right](0.32521)$$

$$= 1.5056 + 0.2422 = 1.7478$$

$$\text{LCL} = \bar{\bar{x}} - A_2\left[\frac{d_2(\text{new})}{d_2(\text{old})}\right]\bar{R}_{\text{old}}$$

$$= 1.5056 - (1.023)\left[\frac{1.693}{2.326}\right](0.32521)$$

$$= 1.5056 - 0.2422 = 1.2634$$

Example continued

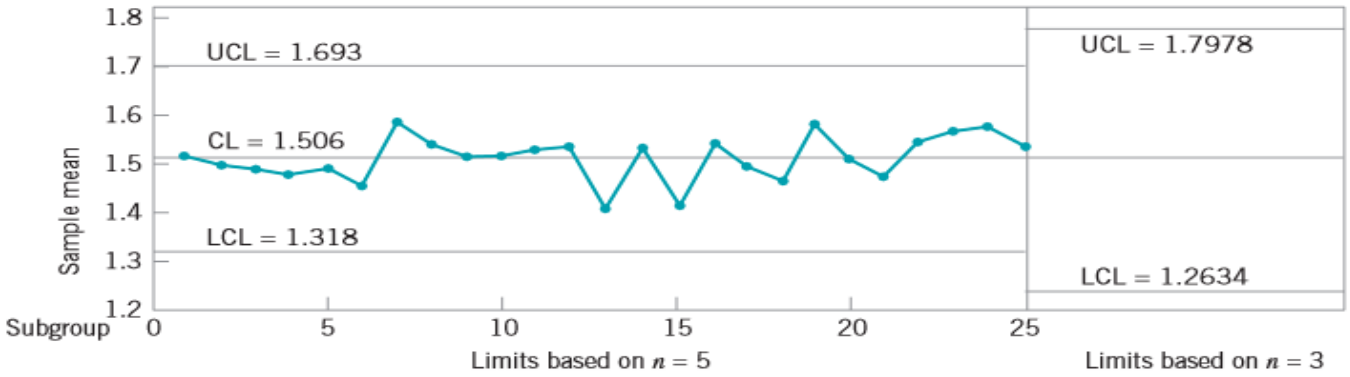
Calculation for R chart

$$\text{UCL} = D_4\left[\frac{d_2(\text{new})}{d_2(\text{old})}\right]\overline{R}_{\text{old}} = (2.574)\left[\frac{1.693}{2.326}\right](0.32521) = 0.6093$$

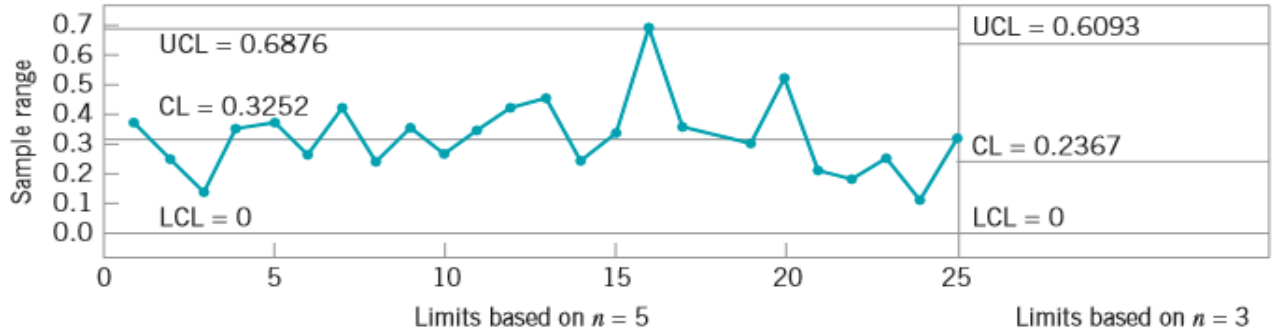
$$\text{LCL} = \max\left\{0, D_3\left[\frac{d_2(\text{new})}{d_2(\text{old})}\right]\overline{R}_{\text{old}}\right\} = 0$$

$$\text{CL} = \overline{R}_{\text{new}} = \left[\frac{d_2(\text{new})}{d_2(\text{old})}\right]\overline{R}_{\text{old}} = \left[\frac{1.693}{2.326}\right](0.32521) = 0.2367$$

$\overline{x}$ chart



R chart



Probability limits on $\bar{x}$ chart

- Control limits in the charts are often expressed in terms of multiples of standard deviation

- It is also possible to define the control limits by specifying the type I error level for the test
  - These are called probability limits for control charts

Probability limits on $\bar{x}$ chart

- Assuming x is normally distributed
  - We may obtain a desired type I error of a by choosing the multiple of sigma for the control limit as $k = Z_{\alpha/2}$, where $Z_{\alpha/2}$ is the upper

  α/2 percentage point of the standard normal distribution
  - If we choose $\alpha = 0.002$, then $Z_{\alpha/2} = Z_{0.001} = 3.09$
  - There is very little difference between the control limits and the probability

$\bar{x}$ and R chart when μ and σ are known

- The $\bar{x}$ chart

$$\text{UCL} = \mu + 3\frac{\sigma}{\sqrt{n}}$$
$$\text{Center line} = \mu$$
$$\text{LCL} = \mu - 3\frac{\sigma}{\sqrt{n}}$$

- Defining $3/\sqrt{n}$ as A, we get

$$\text{UCL} = \mu + A\sigma$$
$$\text{Center line} = \mu$$
$$\text{LCL} = \mu - A\sigma$$

- For R chart

$$\text{UCL} = d_2\sigma + 3d_3\sigma$$
$$\text{Center line} = d_2\sigma$$
$$\text{LCL} = d_2\sigma - 3d_3\sigma$$

- Defining the constants $D_1 = d_2 - 3d_3$, $D_2 = d_2 + 3d_3$ we get:

$$\text{UCL} = D_2\sigma$$
$$\text{Center line} = d_2\sigma$$
$$\text{LCL} = D_1\sigma$$

Interpreting the charts

- In interpreting patterns on the chart, we must first determine whether or not the R chart is in control
  - You must eliminate assignable causes from R chart first
  - Never attempt to interpret the x chart when the R chart indicates an out-of-control condition
- Typical patterns may be seen and interpreted from the charts

Cyclic Pattern

- Such a pattern on the chart may result from systematic environmental changes such as temperature, operator fatigue, regular rotation of operators and/or machines, or fluctuation in voltage or pressure or some other variable in the production equipment.
- R charts will sometimes reveal cycles because of maintenance schedules, operator fatigue, or tool wear resulting in excessive variability.

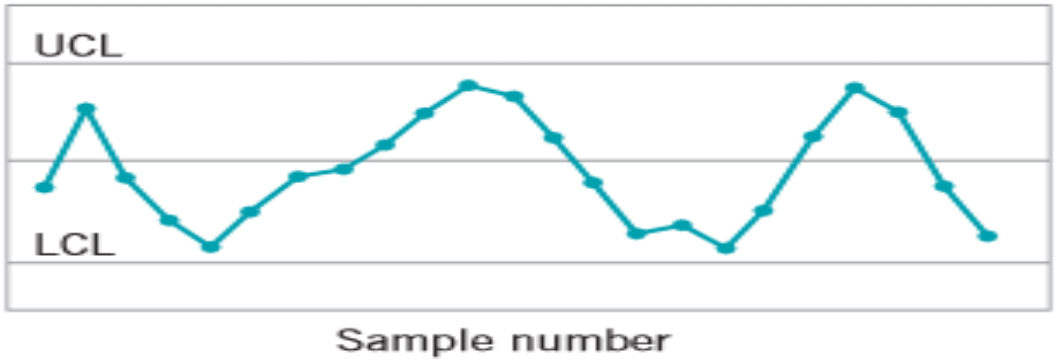


FIGURE 6.8 Cycles on a control chart.

Mixture

- A mixture is indicated when the plotted points tend to fall near or slightly outside the control limits, with relatively few points near the center line
- A mixture pattern can also occur when output product from several sources (such as parallel machines) is fed into a common stream



FIGURE 6.9 A mixture pattern.

Shift in process level

- Shifts may result from the introduction of new workers; changes in methods, raw materials, or machines; a change in the inspection method or standards

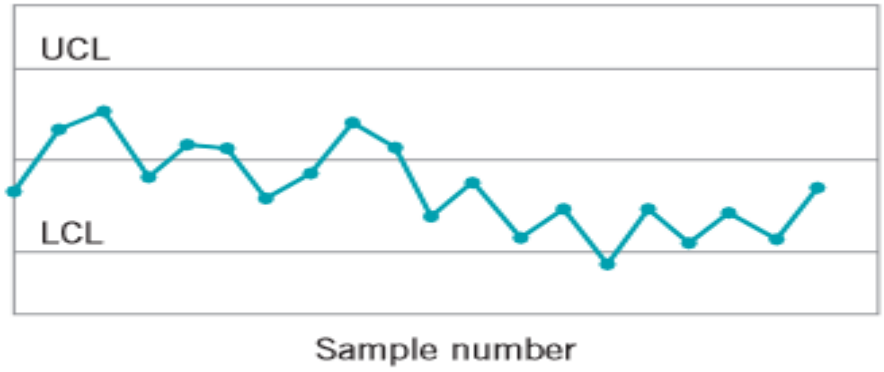


FIGURE 6.10 A shift in process level.

Trend

- Continuous movement in one direction
- Causes-gradual wearing out or deterioration of a tool or some other critical process component, operator fatigue, seasonal influences, such as temperature

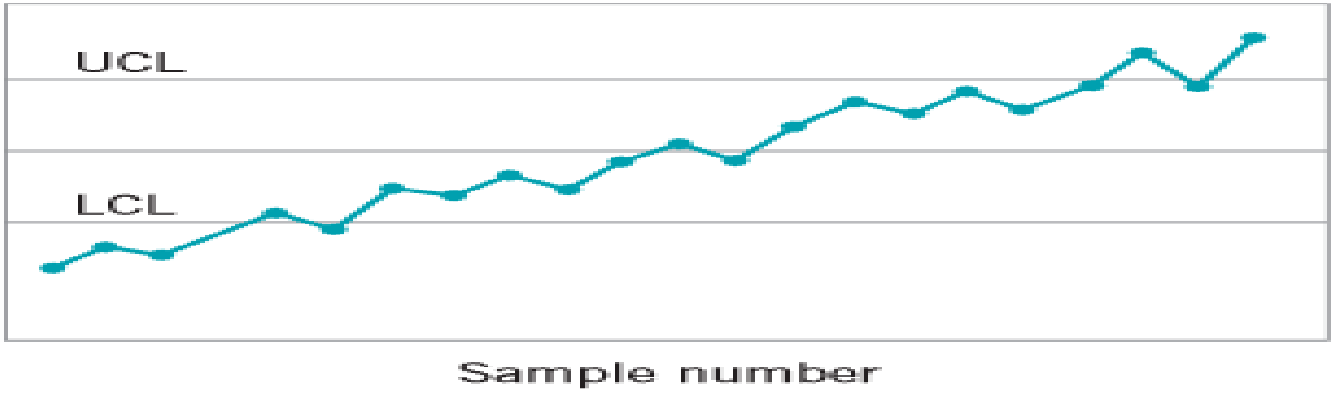


FIGURE 6.11 A trend in process level.

Stratification

- Tendency for the points to cluster artificially around the center line
- incorrect calculation of control limits, sampling process collects one or more units from several different underlying distributions within each subgroup

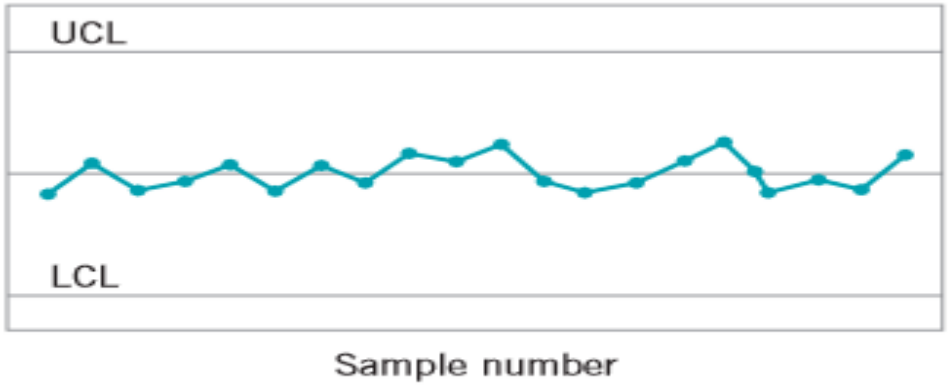


FIGURE 6.12 A stratification pattern.

S chart

- It is occasionally desirable to estimate the process standard deviation directly instead of indirectly through the use of the range R
- S chart is preferred over R chart when
  - The sample size n is moderately large—say, n>10 or 12.
  - The sample size n is variable.

The operating characteristic function

- The ability of the $\bar{x}$ and R charts to detect shifts in process quality is described by their operating characteristic (OC) curves
- For an $\bar{x}$ chart
  - If the mean shifts from the in-control value—say, μ_0 to another value $\mu_1 = \mu_0 + k\sigma$, the probability of not detecting this shift on the first subsequent sample or the β-risk is

$$\beta = P\left\{\text{LCL} \le \bar{x} \le \text{UCL} \middle| \mu = \mu_1 = \mu_0 + k\sigma\right\}$$

Since $\bar{x} \sim N(\mu, \sigma^2/n)$, $\text{UCL} = \mu_0 + L\sigma/\sqrt{n}$ and $\text{LCL} = \mu_0 - L\sigma/\sqrt{n}$,

$$\beta = \Phi\left(L - k\sqrt{n}\right) - \Phi\left(-L - k\sqrt{n}\right)$$

OC curve

- Suppose that we are using an $\bar{x}$ chart with L = 3 (the usual three-sigma limits), the sample size n = 5, and we wish to determine the probability of detecting a shift to $\mu_1 = \mu_0 + 2\sigma$ on the first sample following the shift. Then, since L = 3, k = 2, and n = 5, we have

$$\begin{aligned}\beta &= \Phi\left[3-2\sqrt{5}\right]-\Phi\left[-3-2\sqrt{5}\right]\\ &= \Phi(-1.47)-\Phi(-7.37)\\ &\cong 0.0708\end{aligned}$$

  - This is the probability of not detecting the shift

OC curve

- The probability that such a shift will be detected on the first subsequent sample is $1 - \beta = 1 - 0.0708 = 0.9292$
- The probability that the shift will be detected on the r^{th} subsequent sample is simply $1 - \beta$ times the probability of not detecting the shift on each of the initial $r - 1$ samples

$$\beta^{r-1}(1-\beta)$$

- The expected number of samples taken before the shift is detected is simply the average run length

$$\text{ARL} = \sum_{r=1}^{\infty} r\beta^{r-1}(1-\beta) = \frac{1}{1-\beta}$$

- For the example

$$\text{ARL} = \frac{1}{1-\beta} = \frac{1}{0.25} = 4$$

Calculating parameters of $\bar{x}$ and s chart

- When σ is known

$$\text{UCL} = B_6\sigma$$
$$\text{Center line} = c_4\sigma$$
$$\text{LCL} = B_5\sigma$$

- When σ is not known, we use s which is the average s of m preliminary sample available

$$\text{UCL} = B_4\bar{s}$$
$$\text{Center line} = \bar{s}$$
$$\text{LCL} = B_3\bar{s}$$

For $\bar{x}$ chart, the parameters become

$$\text{UCL} = \bar{\bar{x}} + A_3\bar{s}$$
$$\text{Center line} = \bar{\bar{x}}$$
$$\text{LCL} = \bar{\bar{x}} - A_3\bar{s}$$

Example

Construct and interpret $\bar{x}$ and s charts using the piston ring inside diameter measurements in Table 6.3.

■ TABLE 6.3

Inside Diameter Measurements (mm) for Automobile Engine Piston Rings

Sample Number	Observations					$\bar{x}_i$	s_i
1	74.030	74.002	74.019	73.992	74.008	74.010	0.0148
2	73.995	73.992	74.001	74.011	74.004	74.001	0.0075
3	73.988	74.024	74.021	74.005	74.002	74.008	0.0147
4	74.002	73.996	73.993	74.015	74.009	74.003	0.0091
5	73.992	74.007	74.015	73.989	74.014	74.003	0.0122
6	74.009	73.994	73.997	73.985	73.993	73.996	0.0087
7	73.995	74.006	73.994	74.000	74.005	74.000	0.0055
8	73.985	74.003	73.993	74.015	73.988	73.997	0.0123
9	74.008	73.995	74.009	74.005	74.004	74.004	0.0055
10	73.998	74.000	73.990	74.007	73.995	73.998	0.0063
11	73.994	73.998	73.994	73.995	73.990	73.994	0.0029
12	74.004	74.000	74.007	74.000	73.996	74.001	0.0042
13	73.983	74.002	73.998	73.997	74.012	73.998	0.0105
14	74.006	73.967	73.994	74.000	73.984	73.990	0.0153
15	74.012	74.014	73.998	73.999	74.007	74.006	0.0073
16	74.000	73.984	74.005	73.998	73.996	73.997	0.0078
17	73.994	74.012	73.986	74.005	74.007	74.001	0.0106
18	74.006	74.010	74.018	74.003	74.000	74.007	0.0070
19	73.984	74.002	74.003	74.005	73.997	73.998	0.0085
20	74.000	74.010	74.013	74.020	74.003	74.009	0.0080
21	73.982	74.001	74.015	74.005	73.996	74.000	0.0122
22	74.004	73.999	73.990	74.006	74.009	74.002	0.0074
23	74.010	73.989	73.990	74.009	74.014	74.002	0.0119
24	74.015	74.008	73.993	74.000	74.010	74.005	0.0087
25	73.982	73.984	73.995	74.017	74.013	73.998	0.0162
						$\Sigma =$ 1850.028	0.2351
						$\bar{\bar{x}} =$ 74.001	$\bar{s} =$ 0.0094

Solution

- Calculate average mean and average Standard Deviation

$$\bar{\bar{x}} = \frac{1}{25}\sum_{i=1}^{25}\bar{x}_i = \frac{1}{25}(1850.028) = 74.001$$

$$\bar{s} = \frac{1}{25}\sum_{i=1}^{25}s_i = \frac{1}{25}(0.2351) = 0.0094$$

- Parameters for x chart

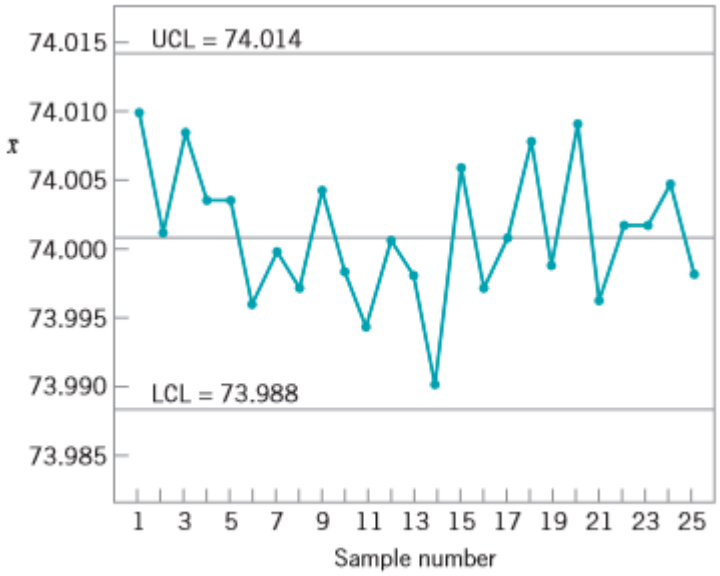
$$\text{UCL} = \bar{\bar{x}} + A_3\bar{s} = 74.001 + (1.427)(0.0094) = 74.014$$
$$\text{CL} = \bar{\bar{x}} = 74.001$$
$$\text{LCL} = \bar{\bar{x}} - A_3\bar{s} = 74.001 - (1.427)(0.0094) = 73.988$$

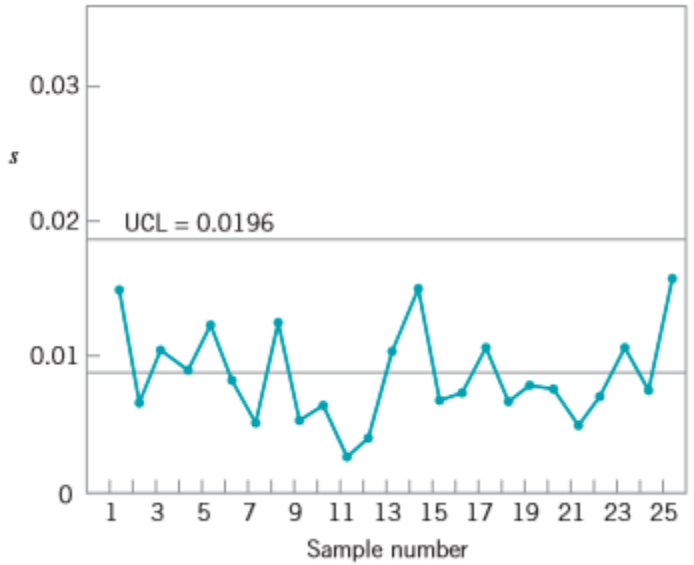
- Parameters for s chart

$$\text{UCL} = B_4\bar{s} = (2.089)(0.0094) = 0.0196$$
$$\text{CL} = \bar{s} = 0.0094$$
$$\text{LCL} = B_3\bar{s} = (0)(0.0094) = 0$$

The charts and process SD estimation

- The x control chart
- The s control chart





$$\hat{\sigma} = \frac{\bar{s}}{c_4} = \frac{0.0094}{0.9400} = 0.01$$

The s^2 chart

- This is based on sample variance

$$\text{UCL} = \frac{\bar{s}^2}{n-1}\chi^2_{\alpha/2,n-1}$$

$$\text{Center line} = \bar{s}^2$$

$$\text{LCL} = \frac{\bar{s}^2}{n-1}\chi^2_{1-(\alpha/2),n-1}$$

- Here s^2 is the average sample variance
- X^2 refers to chi-square distribution
  - Look at the degrees of freedom
- This chart has its limits defined in terms of probability

Moving Range Control Chart

- In many cases sample size used for process monitoring is 1
  - Repeat measurements on the process differ only because of laboratory or analysis error, as in many chemical processes
  - Multiple measurements are taken on the same unit of product, such as measuring oxide thickness at several different locations on a wafer in semiconductor manufacturing.
- In such situations, the control chart for individual units is useful
- We use the moving range two successive observations as the basis of estimating the process variability

$$\mathrm{MR}_i = |x_i - x_{i-1}|$$

Example

The mortgage loan processing unit of a bank monitors the costs of processing loan applications. The quantity tracked is the average weekly processing costs, obtained by dividing total weekly costs by the number of loans processed during the week. The processing costs for the most recent 20 weeks are shown in Table 6.6. Set up individual and moving range control charts for these data.

$$\text{UCL} = \bar{x} + 3\frac{\overline{\text{MR}}}{d_2}$$

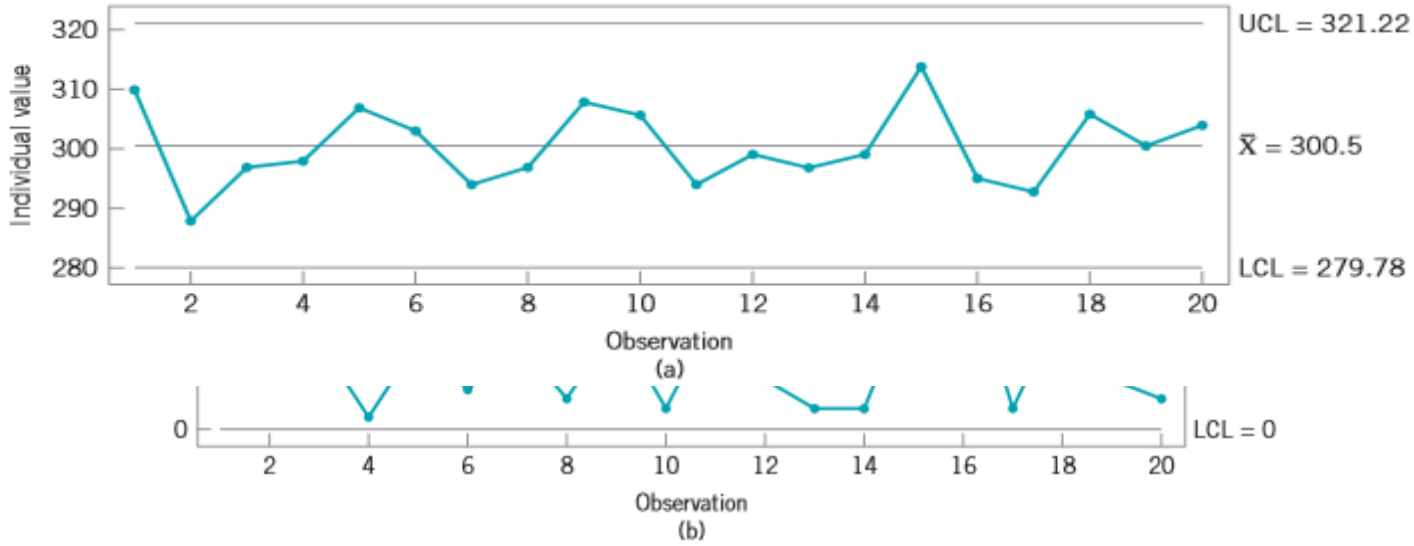
$$\text{Center line} = \bar{x}$$

$$\text{LCL} = \bar{x} - 3\frac{\overline{\text{MR}}}{d_2}$$

Costs of Processing Mortgage Loan Applications

Weeks	Cost x	Moving Range MR
1	310	
2	288	22
3	297	9
4	298	1
5	307	9
6	303	4
7	294	9
8	297	3
9	308	11
10	306	2
11	294	12
12	299	5
13	297	2
14	299	2
15	314	15
16	295	19
17	293	2
18	306	13
19	301	5
20	304	3
	$\bar{x} = 300.5$	$\overline{\text{MR}} = 7.79$

Charts


Individual value
320
310
300
290
280
UCL = 321.22
$\bar{X}$ = 300.5
LCL = 279.78
2
4
6
8
10
12
14
16
18
20
Observation
(a)
0
LCL = 0
2
4
6
8
10
12
14
16
18
20
Observation
(b)

Normality assumption before using the MR chart

- If the process shows evidence of even moderate departure from normality, the control limits calculated may be entirely inappropriate
- One approach to dealing with the problem of non-normality would be to determine the control limits for the individuals control chart based on the percentiles of the correct underlying distribution
- These percentiles could be obtained from a histogram if a large sample (at least 100 but preferably 200 observations) were available, or from a probability distribution fit to the data
- Before plotting a MR chart, it is better to check for normality using histogram or normal probability plot

R Example

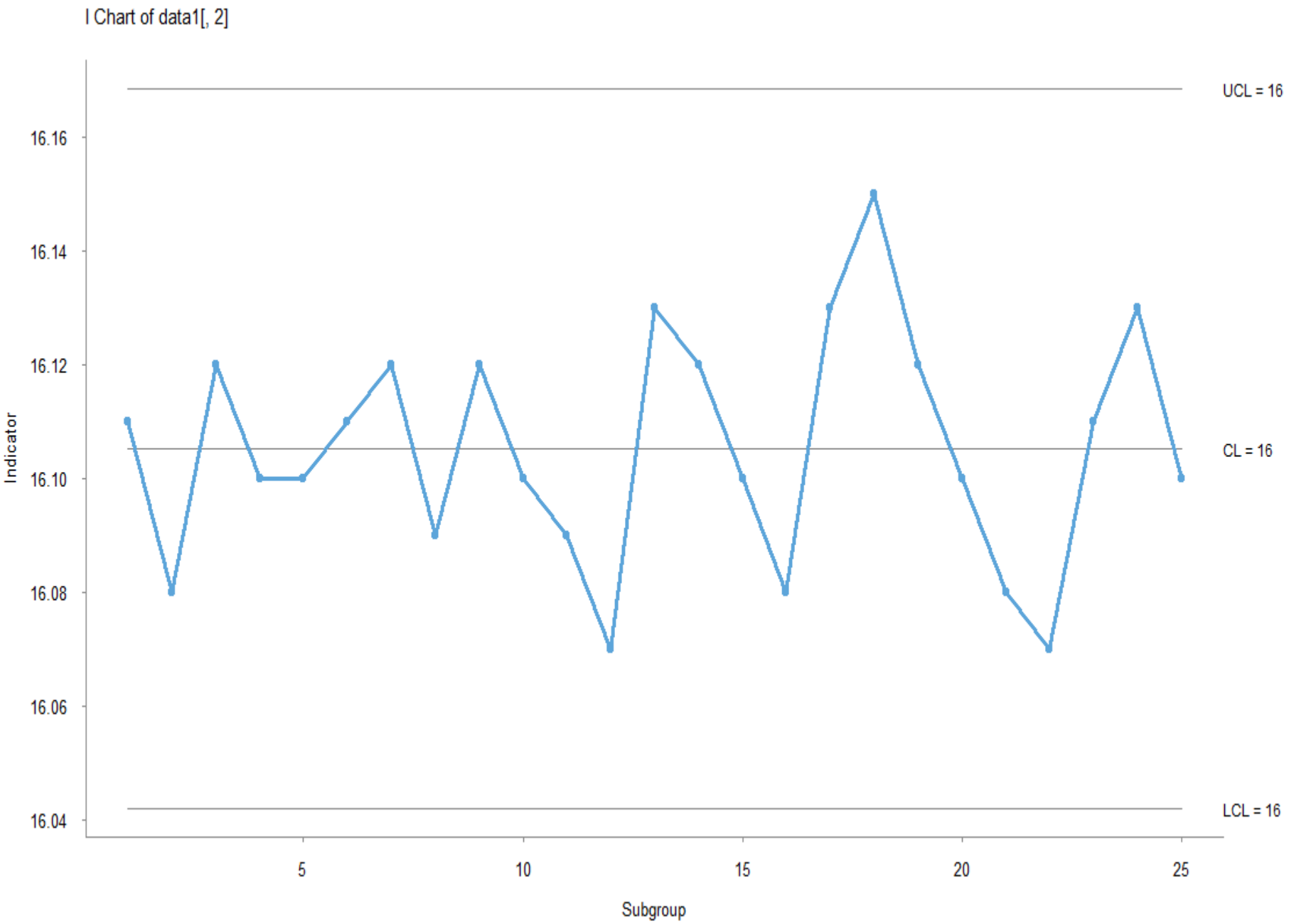
6.53. One-pound coffee cans are filled by a machine, sealed, and then weighed automatically. After adjusting for the weight of the can, any package that weighs less than 16 oz is cut out of the conveyor. The weights of 25 successive cans are shown in Table 6E.19. Set up a moving range control chart and a control chart for individuals. Estimate the mean and standard deviation of the amount of coffee packed in each can. Is it reasonable to assume that can weight is normally distributed? If the process remains in control at this level, what percentage of cans will be underfilled?

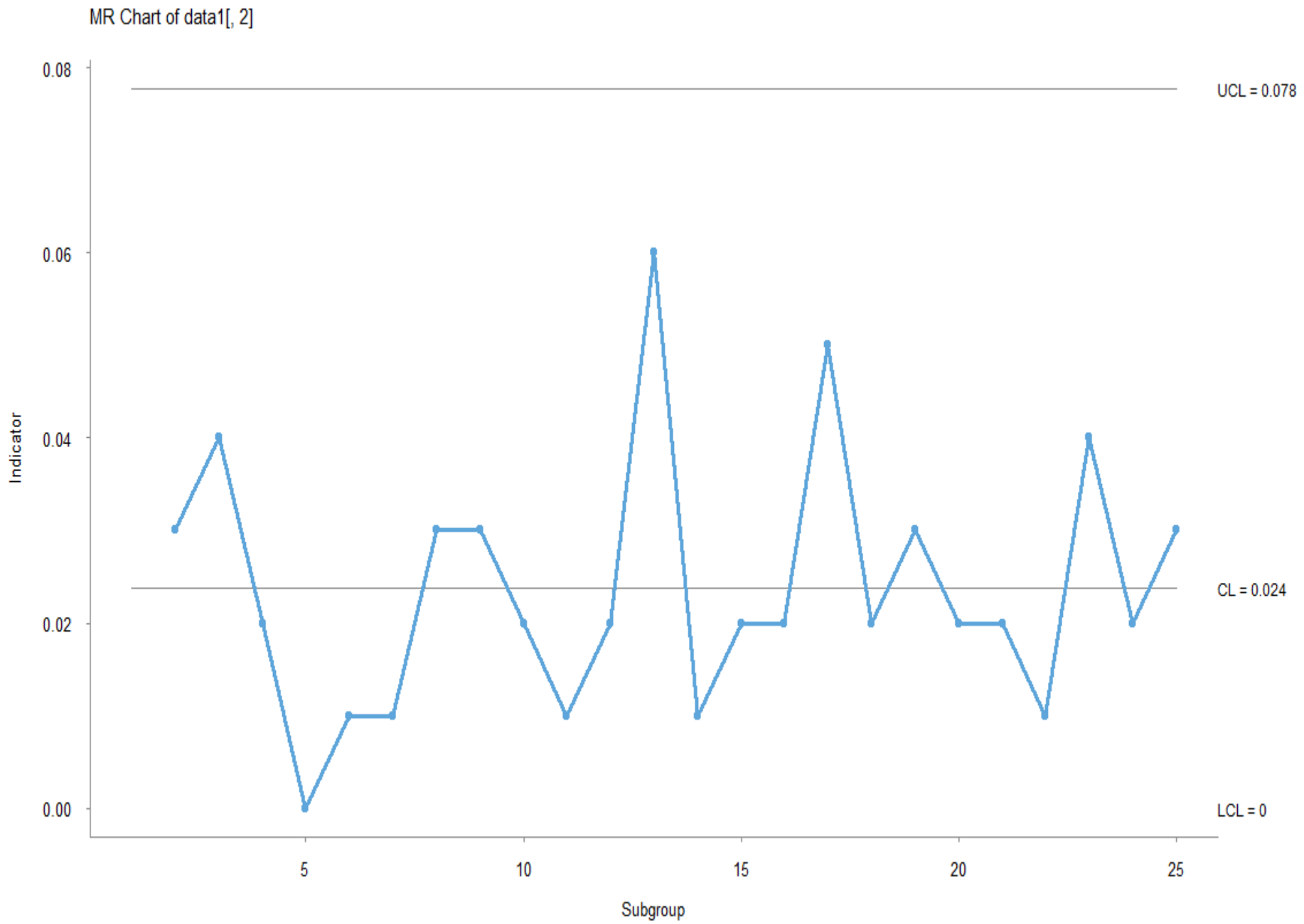
TABLE 6E.19
Can Weight Data for Exercise 6.53

Can Number	Weight	Can Number	Weight
1	16.11	14	16.12
2	16.08	15	16.10
3	16.12	16	16.08
4	16.10	17	16.13
5	16.10	18	16.15
6	16.11	19	16.12
7	16.12	20	16.10
8	16.09	21	16.08
9	16.12	22	16.07
10	16.10	23	16.11
11	16.09	24	16.13
12	16.07	25	16.10
13	16.13		

R code

- data1<-matrix(nrow=25,ncol=3)
- data1[,2]<-c(16.11,16.08,16.12,16.10,16.10,16.11,16.12,16.09,16.12,16.10,16.09,16.07,16.13,16.12,16.10,16.08,16.13,16.15,16.12,16.10,16.08,16.07,16.11,16.13,16.10)
- library(qicharts) # to plot the MR and I chart we use qicharts package
  - The package must be installed prior to it usage
- i<-qic(data1[,2],chart="i")
- j<-qic(data1[,2],chart="mr")
- Keep in mind in the I chart the control limits are rounded off
  - For example i$cl gives 16.02 while it is shown 16 in the graph


I Chart of data1[, 2]
UCL = 16
CL = 16
LCL = 16
16.16
16.14
16.12
16.10
16.08
16.06
16.04
Indicator
5
10
15
20
25
Subgroup


MR Chart of data1[, 2]
0.08
0.06
0.04
0.02
0.00
Indicator
UCL = 0.078
CL = 0.024
LCL = 0
5
10
15
20
25
Subgroup

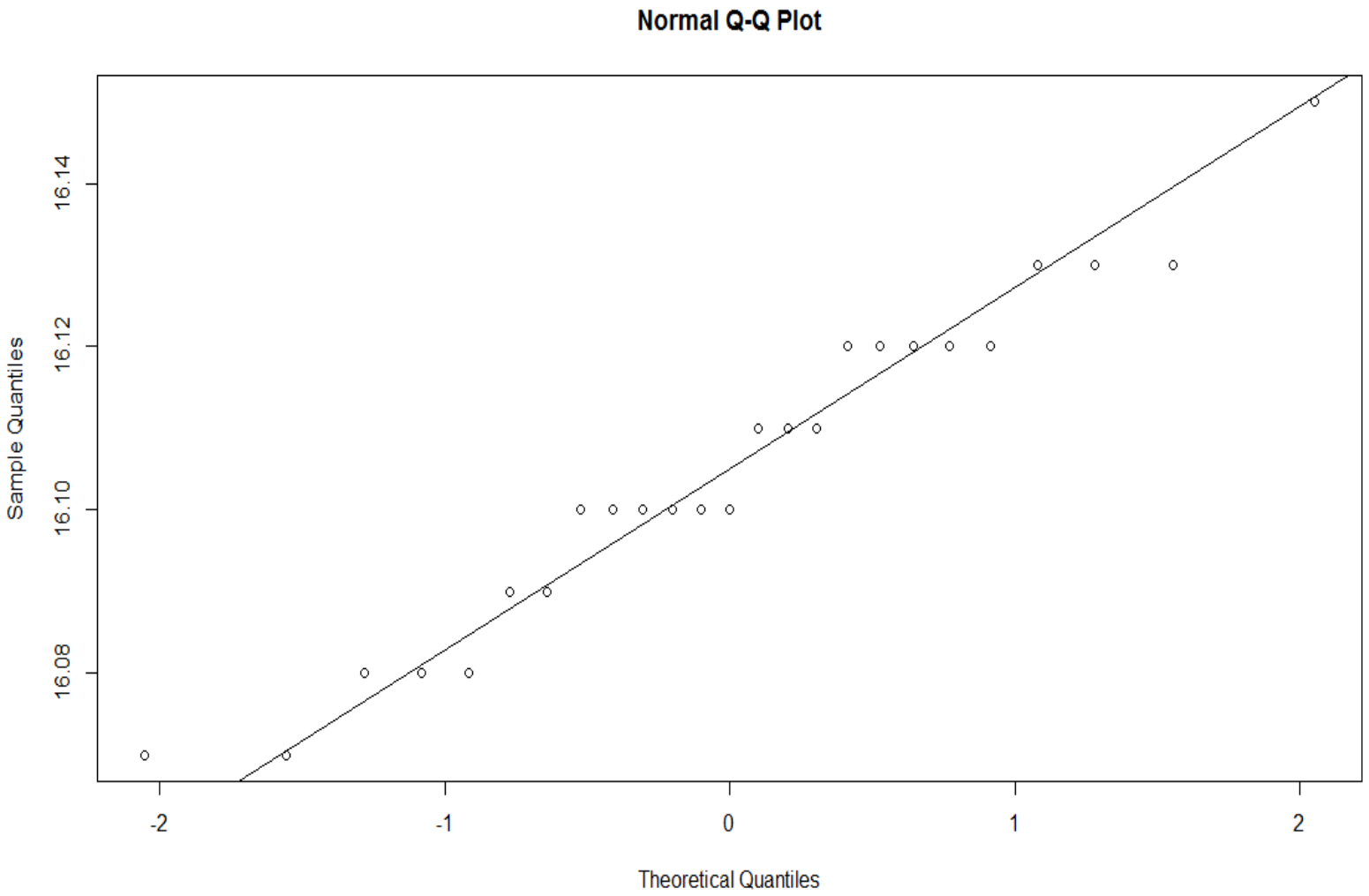
Calculations

- Mean
  - i$cl=16.052
  - Standard deviation estimation

$$\hat{\sigma}_1 = 0.8865\overline{\mathrm{MR}}$$

  - Here SD=.8865*.02375=.0210
    - .02375 is obtained by j$cl
- To check normality we again use the qqplot
- qqnorm(data1[,2])
- qqline(data1[,2])

Normal Q-Q Plot



Percentage of cans under filled

- % cans under filled can be estimated from the lower control limit
- For a can to remain under filed, the weight must be below the lower control limit.
- So if we can find the probability of cans having weight below the lower control limit, we should know the %of cans to be under filled
- Pr(x<16.04204)=(16.04204-16.052)/.0210=.317

References

- http://data.library.virginia.edu/understanding-q-q-plots/